physically measures the viscous length K_v relative to the particle length h^*. It is 0(1) in this example, thus fulfilling the continuum assumption, crucial near shocks. Finally, we indicate that increased values of the viscosity K_v speed convergence (because smoother gradients appear), but these tend also to smear out the shock. Surface pressures ($C_p = -0.231\ \varphi_x$) were calculated by second-order difference formulas.

The preceding problem was run on the CDC 7600 Cybernet System. Convergence was achieved after 500 sweeps of the flowfield, such that surface C_p's differed less than ½%/50 sweeps. Convergence time was estimated at 200 sec. Figure 2 compares the present Mach 0.820 results with 0.825 results for the same airfoil obtained by Martin's[1] improved Murman-Cole algorithm. We note how the extrapolated agreement is excellent, our results being somewhat less peaky, as expected. The magnitude and position of shock jumps and shape of the supersonic zone appear to be well predicted. The shock is smeared over two meshwidths, but this can be reduced by decreasing K_v (at the expense of increasing convergence time).

Discussion

The results obtained from the second-order scheme suggest that solutions obtained in the limit $K_v \to 0$ contain the class of inviscid solutions under Murman-Cole-type jump conditions. The present method, as it stands, however, is not as efficient. The primary reason is the use of Keller's one-dimensional box scheme, which *triples* the number of dependent variables. We thus are pursuing currently this "direct viscosity approach" using a one-dependent-variable second-order differencing scheme. A number of advantages, though, are obvious. Because of second-order accuracy, a coarser mesh can be used to produce Murman-Cole-type results, which are first-order accurate. Secondly, Richardson's extrapolation is possible[8]; thus, good results can be obtained from two relatively coarse nets. Thirdly, the explicit presence of a viscous term permits treatment of non-Rankine-Hugoniot jumps. The major selling point, however, seems to be the simplified program logic. Thus, straightforward extension to the full potential equation with, for example, a simple streamwise diffusion term should be possible. Use of the box scheme in turning-point problems is not new. Our application is analogous to accounting for 0(1) viscous effects near critical layers in direct solutions to the Orr-Sommerfeld equation.

Acknowledgments

The author wishes to thank K. C. Reddy for a number of ideas and for access to some early results. Thanks also go to A. Jameson for some stimulating discussions.

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Nonstationarity in Gun Tunnel Flows

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Nomenclature

$\tilde{e}$ = rms value of the fluctuating voltage
e_i = $\tilde{e}$ at any instant a function of time
$\bar{e}$ = mean value of e_i over a sampling period
f = frequency
p_0 = stagnation pressure
p_{41} = gun tunnel driver pressure ratio
T = sampling period
T_r = wire recovery temperature
T_W = hot-wire temperature
t = time measured from the beginning of the run
v^* = virtual sensitivity of the hot-wire
ϵ_n = nonstationary error
ϵ_r = random error
σ_r = standard deviation due to randomness
σ_n = standard deviation due to nonstationarity
Δe_m = hot-wire mass flow sensitivity
Δe_t = hot-wire total temperature sensitivity
Δf = bandwidth

Introduction

TEST facilities such as shock tubes and gun tunnels have been used in recent years for unsteady flow measurement.[1-4] These facilities suffer from the fact that their flows are transient as well as nonstationary; that is, fluctuation signal spectra vary with time. Statistical analysis of the flow unsteadiness in these facilities would involve both the random error due to the statistical averaging process and a nonstationarity error due to the time dependence of the flow. For an analysis of these unsteady flow signals it is necessary to know 1) whether an optimum sampling period exists during the run time for which the total error is within an acceptable limit, and 2) whether a meaningful mean flow condition can be assigned for that sampling period. This Note enlightens these problems in a hypersonic gun tunnel flow.

Tests

Hot-wire anemometer studies were carried out at a point in the exit core flow of a Mach 7 hypersonic jet in a gun tunnel at Loughborough University of Technology. Tests were performed for four wire overheats at $p_0 = 610$ psia and $p_{41} = 10$. The mean voltages were recorded on oscillograms and the fluctuation voltages on a DR channel of an Epsylon MR1200 tape recorder. The DR frequency response was 200 Hz-80 KHz.

Analysis

Recovery temperature during the runs were computed from the hot-wire voltages by the method suggested by Vrebalovich.[5] The extent of nonstationarity in the flow can be seen from the results shown in Fig. 1. The mean tem-

Received March 29, 1977; revision received May 31, 1977.

Index categories: Supersonic and Hypersonic Flow; Nonsteady Aerodynamics.

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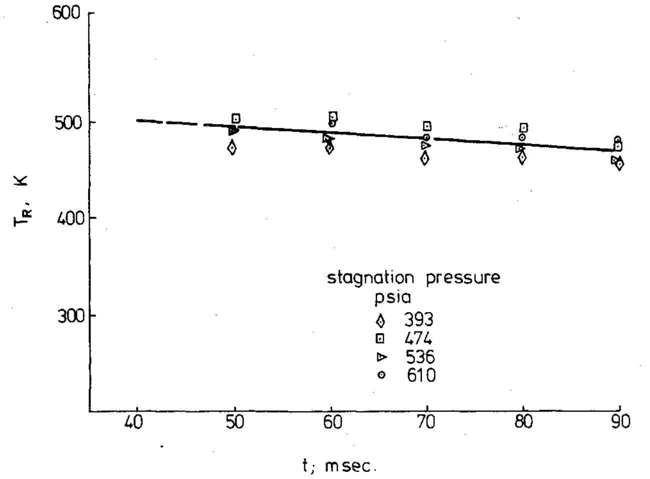

Fig. 1 Variation of the flow recovery temperature during the run time.

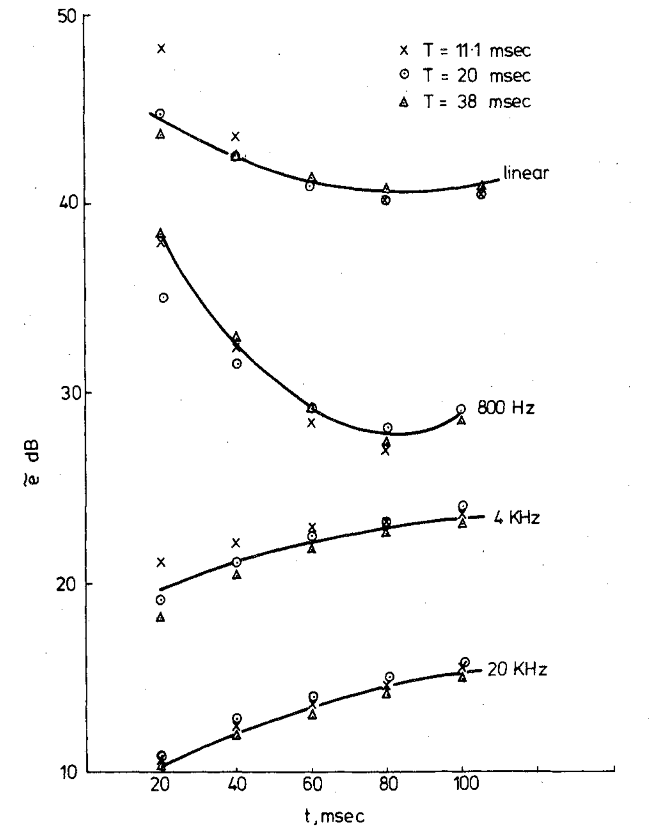

Fig. 2 Variation of the rms value of the hot-wire signal with the run time for various sampling periods and frequencies; $p_0 = 610$ psia, $T_W = 585$ K.

peraure history indicates a temperature drop of about 1000 K/sec. This compares with a drop of 2500 K/sec measured in the University of Southampton gun tunnel.[6] The measured Mach number and stagnation pressure were constant during the run, therefore there would be corresponding changes in the density and flow velocity.

Third-octave and linear analyses of the fluctuation signals were carried out on an B&K analog system at a tape playback speed of one-eighth the recording speed. Mean traces of the $\bar{e} \sim t$ histories were drawn on the paper level recordings. This was repeated for a number of averaging (sampling) periods until a convergence was obtained between the mean traces at suitable averaging periods. This procedure is equivalent to obtaining the mean level variation by statistical regression analysis. The converged traces then represent the variation in $\bar{e}$ for a given p_0 and T_W. Typical results for $p_0 = 610$ psia and $T_W 585$ K are shown in Fig. 1. The lack of convergence during the initial part of the run can be attributed to the high nonstationarity present during that period.

The nonstationarity error ϵ_n can be defined as

$$\epsilon_n{}^2 = \sum_{i=1}^{N} \frac{(e_i - \bar{e})^2}{n-1}$$

where e_i is the instantaneous rms value of the fluctuating voltage, a function of time. $\bar{e}$ is the mean value of e_i over a sampling period T, thus

$$\bar{e} = \frac{\sum_{i=1}^{N} e_i}{N}$$

and N is the number of observations of e_i over the sampling period. The standard deviation σ_n of ϵ_n, in dB, can be defined as

$$\sigma_n{}^2 = 10 \log \left(1 + (\epsilon_n{}^2 / \bar{e}^2)\right)$$

The normalized random error or the error due to uncertainty can be defined as

$$\epsilon_r = 1/\sqrt{T\Delta f}$$

The standard deviation σ_r of ϵ_r, in dB, can be defined as

$$\sigma_r{}^2 = 10 \log [1 + (1/T\Delta f)]$$

The total standard deviation σ_T due to both types of error is then given by

$$\sigma_T{}^2 = \sigma_n{}^2 + \sigma_r{}^2$$

The variation of σ_T with T for $p_0 = 610$ psia and $T_W = 585$ K is shown in Fig. 3a. Here the sampling commences at the 20 msec instant. If one chooses a maximum allowable standard deviation of 1 dB, then it is possible to choose sampling

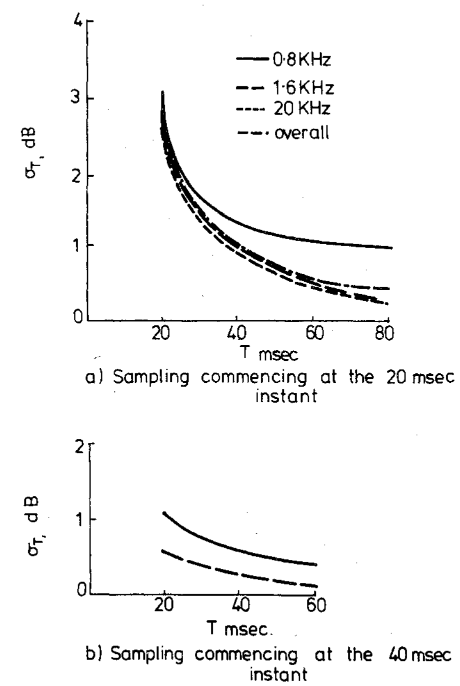

Fig. 3 Influence of the sampling time on the standard deviations.

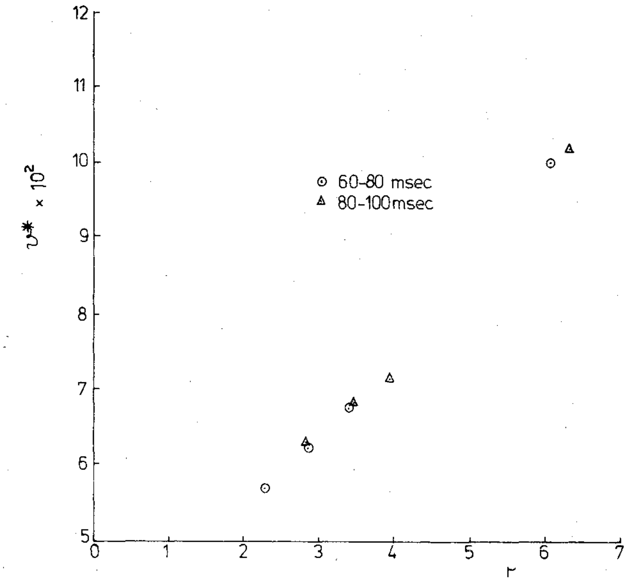

Fig. 4 Kovasznay mode diagram of the hot-wire signal; $P_0 = 610$ psia, $T = 20$ msec.

periods at frequencies ≥ 1.6 KHz within the run time, but for the frequency of 0.8 KHz the standard deviation is greater than 1 dB throughout the run. It is also observed that, for frequencies ≥ 1.6 KHz, the variation of standard deviation with frequency is small. If, however, the sampling commences at the 40 msec instant, then the standard deviation, in general, decreases to those shown in Fig. 3b, and it is possible to analyze the 0.8 KHz case within the specified limit of 1 dB. This implies that the nonstationarity decreases as the run progresses, and it would be more appropriate to sample toward the end of the run.

The overall sampling period of 20 msec used on the results of Fig. 3b would result in a total temperature, density, and velocity changes of 3%, 3%, and 1½%, respectively, for the interval of the run 40-60 msec. For changes of these magnitudes it would prove possible to assign meaningful values to the mean flow conditions.

A Kovasznay mode analysis[7] of the hot-wire signals was made with a sampling period of $T = 20$ msec. The results for sampling commencing at the 60 and 80 msec instants are shown in Fig. 4. The results include only the frequencies > 1600 HZ. The mode diagrams are linear with a positive slope. This means that the correlation between the mean flow (the slope of the line) and total temperature fluctuations (the intercept on the v^* axis) is -1. A correlation of -1 has two possibilities: 1) entropy fluctuations for which the straight line would have an intercept of $r = -[1 + (\gamma - 1)/2)M_\infty^2]^{-1}$ on the r axis, which is not the case here; or 2) a pure sound field with a finite total temperature fluctuation implying that sound originates from a moving source. Construction of a mode diagram from the data taken from different runs may be questionable. Nevertheless, the data indicate that the differences in the magnitude of the fluctuations between 60 and 100 msec is negligible and the disturbance field appears to be essentially a sound made of disturbance as in supersonic wind tunnel flows.[8,9]

Conclusion

It appears that the nonstationary flows encountered in short duration test facilities can be used for the study of unsteady flow problems, provided care is taken in selecting the sampling period and the instant from which the sampling commences.

The present tests show that an optimum sampling period of 20 msec toward the end of the run results in a total standard deviation of less than 1 dB and a meaningful mean flow condition can be assigned for this sampling period.

Acknowledgment

This work originated as part of the research supported by SRC Contract No. B/RG/3323:2.

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Flow Characteristics in an Expansion Tunnel as Inferred from Velocity Measurements

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Introduction

An experimental study[1,2] was undertaken recently to determine if predicted[3] advantages of operating the Langley expansion tube[1,4] as an expansion tunnel exist in real life. (The Langley expansion tunnel is simply an expansion tube with a conical nozzle positioned at the exit of the acceleration section; hence, nozzle entrance flow conditions are hypersonic and hypervelocity.) Heretofore, the only flow diagnostic used to deduce flow characteristics in the vicinity of the nozzle exit was vertical surveys and time histories of the pitot pressure. As discussed in Ref. 1, additional measurements are required to define nozzle flow characteristics and determine if nozzle flow conditions are satisfactory for aerodynamic testing. To provide this information, a photoionization technique[5] was employed to measure vertical surveys of the axial component of flow velocity just downstream of the nozzle exit. Preliminary results from these velocity measurements are presented herein, along with estimates of freestream density inferred from the velocity measurement technique. These data are compared to measured pitot pressure; thus, two in-

Received May 5, 1977.

Index categories: Nozzle and Channel Flow; Supersonic and Hypersonic Flow.

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